

Exercise F

1. Prove that $\sqrt{5} - \sqrt{3}$ is not a rational number.
2. Arrange the following in descending order of magnitude: $\sqrt[3]{90}, \sqrt[4]{10}, \sqrt{6}$
3. Simplify the following:

$$(i) (4\sqrt{3} - 2\sqrt{2})(3\sqrt{2} + 4\sqrt{3})$$

$$(ii) (2 + \sqrt{3})(3 + \sqrt{5})$$

$$(iii) (\sqrt{3} + \sqrt{2})^2$$

$$(iv) \left(\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11} \right) + \left(\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11} \right)$$

4. Rationalize the denominator of the following:

$$(i) \frac{2}{\sqrt{3} - \sqrt{5}} \quad (ii) \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \quad (iii) \frac{6}{\sqrt{5} + \sqrt{2}} \quad (iv) \frac{1}{8 + 5\sqrt{2}}$$

$$(v) \frac{3 - 2\sqrt{2}}{3 + 2\sqrt{2}} \quad (vi) \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \quad (vii) \frac{4}{\sqrt{7} + \sqrt{3}} \quad (viii) \frac{1}{5 + 3\sqrt{2}}$$

5. Rationalise the denominator of the following:

$$(i) \frac{2}{3\sqrt{3}} \quad (ii) \frac{16}{\sqrt{41} - 5} \quad (iii) \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$(iv) \frac{\sqrt{40}}{\sqrt{3}} \quad (v) \frac{3 + \sqrt{2}}{4\sqrt{2}} \quad (vi) \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$$

$$(vii) \frac{\sqrt{6}}{\sqrt{2} + \sqrt{3}} \quad (viii) \frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \quad (ix) \frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}}$$

6. If $a = 6 - \sqrt{35}$, find the value of $a^2 + \frac{1}{a^2}$.

7. If $x = 3 + \sqrt{8}$, find the value of (i) $x^2 + \frac{1}{x^2}$ and (ii) $x^4 + \frac{1}{x^4}$

8. Simplify, by rationalizing the denominator $\frac{2\sqrt{6}}{\sqrt{2} + \sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6} + \sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6} + \sqrt{2}}$

9. Simplify, by rationalizing the denominator

$$\frac{1}{3 - \sqrt{8}} - \frac{1}{\sqrt{8} - \sqrt{7}} + \frac{1}{\sqrt{7} - \sqrt{6}} - \frac{1}{\sqrt{6} - \sqrt{5}} + \frac{1}{\sqrt{5} - 2}$$

10. If $x = \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$ and $y = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$, find the value of $x^2 + y^2 + xy$.

11. If $x = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ and $y = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$, find the value of $x^2 + y^2$.

12. If $x = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$ and $y = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$, find the value of $x + y + xy$.

13. If $x = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $y = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, find the value of $x^2 - y^2$.
14. If $\frac{5+2\sqrt{3}}{7+\sqrt{3}} = a - \sqrt{3}b$, find a and b where a and b are rational numbers.
15. If a and b are rational numbers and $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$, find the values of a and b.
16. If a and b are rational numbers and $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$, find the values of a and b.
17. If a and b are rational numbers and $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$, find the values of a and b.
18. Evaluate: $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{9}+\sqrt{8}}$
19. If $x = \frac{1}{2+\sqrt{3}}$, find the value of $2x^3 - 7x^2 - 2x + 1$.
20. If $x = \frac{1}{2-\sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 5$.
21. If $\sqrt{2} = 1.414$ and $\sqrt{5} = 2.236$, find the value of $\frac{\sqrt{10}-\sqrt{5}}{2\sqrt{2}}$ upto three places of decimals.
22. Find six rational numbers between 3 and 4.
23. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.
24. Find the value of a and b in $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$.
25. Find the value of a and b in $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$.
26. Find the value of a and b in $\frac{5-\sqrt{6}}{5+\sqrt{6}} = a - b\sqrt{6}$.
27. Simplify $\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$ by rationalizing the denominator.
28. Simplify $\frac{\sqrt{5}-1}{\sqrt{5}+1} + \frac{\sqrt{5}+1}{\sqrt{5}-1}$ by rationalizing the denominator.
29. Simplify $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ by rationalizing the denominator.
30. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, find (i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$.
31. If $x = 4 - \sqrt{15}$, find (i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$.
32. If $x = 2 + \sqrt{3}$, find (i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$.
33. Represent the real number $\sqrt{10}$ on the number line.
34. Represent the real number $\sqrt{13}$ on the number line.

35. Represent the real number $\sqrt{7}$ on the number line.
36. Represent the real number $\sqrt{2}, \sqrt{3}, \sqrt{5}$ on a single number line.
37. Find two rational number and two irrational number between $\sqrt{2}$ and $\sqrt{3}$.
38. Find the decimal expansions of $\frac{10}{3}, \frac{7}{8}$ and $\frac{1}{7}$.
39. Show that 3.142678 is a rational number. In other words, express 3.142678 in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
40. Show that 0.3333..... can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
41. Show that 1.27272727..... can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
42. Show that 0.23535353..... can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
43. Express the following in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
- (i) $0.\bar{6}$ (ii) $0.4\bar{7}$ (iii) $0.00\bar{1}$ (iv) $0.2\bar{6}$
44. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.
45. Visualize the representation of $5.3\bar{7}$ using successive magnification
46. Visualize $4.\bar{26}$ on the number line, using successive magnification upto 4 decimal places.
47. Visualize 3.765 on the number line, using successive magnification.
48. Find the value of a and b in each of the following:
- (i) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$ (ii) $\frac{3+\sqrt{7}}{3-\sqrt{7}} = a+b\sqrt{7}$ (iii) $\frac{7+\sqrt{5}}{7-\sqrt{5}} = a+b\sqrt{5}$
49. Simplify each of the following by rationalizing the denominator.
- (i) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ (ii) $\frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}+2}{\sqrt{5}-2}$
50. Evaluate the following expressions:
- (i) $\left(\frac{256}{6561}\right)^{\frac{3}{8}}$ (ii) $(15625)^{\frac{1}{6}}$ (iii) $\left(\frac{343}{1331}\right)^{\frac{1}{3}}$
- (iv) $\sqrt[8]{\frac{6561}{65536}}$ (v) $343^{\frac{1}{3}}$
51. Simplify: $\frac{\sqrt{32} + \sqrt{48}}{\sqrt{8} + \sqrt{12}}$
52. Simplify: $\frac{7}{3\sqrt{3} - 2\sqrt{2}}$

53. Simplify: (i) $\sqrt[4]{\sqrt[3]{2^2}}$ (ii) $\sqrt[3]{2} \cdot \sqrt[4]{2} \cdot \sqrt[12]{32}$

54. If $\sqrt{2} = 1.4142$, then find the value of $\frac{\sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}-1}}$.

55. If $\sqrt{3} = 1.732$, then find the value of $\frac{\sqrt{\sqrt{3}+1}}{\sqrt{\sqrt{3}-1}}$.

56. Find the value of a if $\frac{6}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{2} - a\sqrt{3}$

57. Evaluate the following expressions:

(i) $\left(\frac{625}{81}\right)^{\frac{1}{4}}$ (ii) $27^{\frac{2}{3}} \times 27^{\frac{1}{3}} \times 27^{-\frac{4}{3}}$ (iii) $(6.25)^{\frac{3}{2}}$

(iv) $(0.000064)^{\frac{5}{6}}$ (v) $(17^2 - 8^2)^{\frac{1}{2}}$

58. Express $0.\bar{6} + 0.\bar{7} + 0.\bar{47}$ in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

59. Simplify: $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

60. If $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, then find the value of $\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$.

61. Simplify:

(i) $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$ (ii) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$ (iii) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

(iv) $\sqrt[4]{12} \times \sqrt[6]{7}$ (v) $\sqrt[4]{28} \div \sqrt[3]{7}$ (vi) $\sqrt[3]{3} + 2\sqrt{27} + \frac{1}{\sqrt{3}}$

(vii) $(\sqrt{3} - \sqrt{5})^2$ (viii) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

(ix) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$ (x) $\frac{\sqrt[2]{3}}{3} - \frac{\sqrt{3}}{6}$

62. If $a = \frac{3+\sqrt{5}}{2}$ then find the value of $a^2 + \frac{1}{a^2}$.

63. Simplify: $(256)^{\left(-4\frac{-3}{2}\right)}$

64. Find the value of $\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{2}{(243)^{\frac{-1}{5}}}$

65. If $a = 5 + 2\sqrt{6}$ and $b = \frac{1}{a}$ then what will be the value of $a^2 + b^2$?

66. Find the value of a and b in each of the following:

$$(i) \frac{3 - \sqrt{5}}{3 + 2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}$$

$$(ii) \frac{\sqrt{2} + \sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} = 2 - b\sqrt{6}$$

$$(iii) \frac{7 + \sqrt{5}}{7 - \sqrt{5}} - \frac{7 - \sqrt{5}}{7 + \sqrt{5}} = a + \frac{7}{11}b\sqrt{5}$$

67. If $a = 2 + \sqrt{3}$, then find the value of $a - \frac{1}{a}$.

68. Rationalise the denominator in each of the following and hence evaluate by taking $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$, upto three places of decimal.

$$(i) \frac{4}{\sqrt{3}} \quad (ii) \frac{6}{\sqrt{6}} \quad (iii) \frac{\sqrt{10} - \sqrt{5}}{2} \quad (iv) \frac{\sqrt{2}}{2 + \sqrt{2}} \quad (v) \frac{1}{\sqrt{3} + \sqrt{2}}$$

69. Simplify:

$$(i) (1^3 + 2^3 + 3^3)^{\frac{1}{2}} \quad (ii) \left(\frac{3}{5}\right)^4 \left(\frac{8}{5}\right)^{-12} \left(\frac{32}{5}\right)^6 \quad (iii) \left(-\frac{1}{27}\right)^{\frac{-2}{3}}$$

$$(iv) \left[\left((625)^{\frac{-1}{2}} \right)^{\frac{-1}{4}} \right]^2 \quad (v) \frac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{\frac{-1}{3}}} \quad (vi) 64^{\frac{-1}{3}} \left[64^{\frac{1}{3}} - 64^{\frac{2}{3}} \right]$$

$$70. \text{Simplify: } \frac{9^{\frac{1}{3}} \times 27^{\frac{-1}{2}}}{3^6 \times 3^{\frac{-2}{3}}}$$